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Note on Mr. Stone's Theory of the Measure of Time.

By Professor Simon Newcomb.

The discussion with Mr. Stone has now reached a stage where nothing remains to be done on my part except to invite Mr. Stone's attention to the conclusions from his theory, as he states them in the May number of the *Monthly Notices.*

A copy of Le Verrier's *Tables du Soleil* has been in my possession since the year 1859. If, with the tables in this book, I compute the sidereal time of mean moon on 1892, January 6, and thence the tabular right ascension of the Moon from Hansen's tables at the moment of its transit over the meridian of Oxford, on that date, I find a difference between the tabular and observed right ascension of $1^{\mathrm{s}}\cdot 36$. The result will be substantially the same if I use the older tables of Carlini.

Mr. Stone maintains that this result is due to my failure to apply a certain correction to the numbers of Le Verrier's tables, which correction is not a function of anything contained in the tables, or of the epoch of any natural event, but of the date when a certain office, three thousand miles away, began to use the tables in computing the solar ephemeris. In his last note on the subject he gives precision to this conclusion by stating explicitly that if the office in question had not begun the use of the tables until 1892, the result would have been $0^{\mathrm{s}}\cdot 10$, instead of $1^{\mathrm{s}}\cdot 36$. The statement is still further strengthened by his answer to my second question, that $0^{\mathrm{s}}\cdot 10$ would also have been the result if Carlini's tables had been continued to the present time in the *Nautical Almanac* Office, though, if I make the computation with Carlini's tables *now*, I shall get nearly $1^{\mathrm{s}}\cdot 36$, the same as with Le Verrier's tables. It thus seems, according to Mr. Stone's theory, that the results which I now get from tables are different from what they would have been had a change of tables not been made in the *Nautical Almanac* Office.

Note on the Lunar Theory. By Professor Ernest W. Brown, M.A.

The terms in Delaunay's expression for the longitude with arguments $2iD \pm l \pm l'$ (i a positive integer), and coefficients of the form $ee'f(m)$ converge in some cases very slowly as far as the series arranged in powers of m are concerned. I have lately calculated the terms of this form, and have obtained the results exhibited in Column II. of the table given below. The values given have been carefully verified, and are correct to the fourth place of decimals of a second :—

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I. Argument.	II. B.	III. D.	IV.	V.	VI. B−D.
$l+l'$	−110″·1758	−110″·1368	−0″·1213	$\frac{1}{3}$	−0″·0390
$2D-l-l'$	+206·8242	+207·0770	−0·3488	$\frac{2}{3}$	−0·2528
$4D-l-l$	+ 4·0861	+ 4·0312	+0·2074	$\frac{1}{3}$	+0·0549
$6D-l-l$	+ 0·0545	+ 0·0188	+0·0188	...	+0·0357
$8D-l-l'$	+ 0·0006	...	...	...	...
$2D+l+l'$	− 2·6612	− 2·6670	−0·0168	$\frac{2}{9}$	+0·0058
$4D+l+l'$	− 0·0370	− 0·0290	−0·0146	...	−0·0080
$6D+l+l'$	− 0·0004	...	...	...	...
$l-l'$	+149·1996	+148·3768	+0·8490	$\frac{1}{2}$	+0·8228
$2D-l+l'$	− 27·8714	− 29·2689	+1·4294	$\frac{1}{2}$	+1·3975
$4D-l+l'$	− 0·5780	− 0·7446	−0·0178	$\frac{1}{2}$	+0·1666
$6D-l+l'$	− 0·0078	− 0·0040	−0·0040	...	−0·0038
$8D-l+l'$	− 0·0001	...	...	...	...
$2D+l-l'$	+ 13·6294	+ 13·6278	+0·1526	$\frac{2}{5}$	+0·0016
$4D+l-l$	+ 0·2160	+ 0·1722	+0·0717	$\frac{3}{4}$	+0·0438
$6D+l-l'$	+ 0·0027	...	...	...	...

The columns numbered I.–VI. are as follows :—

I. The arguments in Delaunay's notation.

II. The results of my calculations. (These are obtained by using the numerical values of m from the start and inserting Delaunay's values of e and e' at the finish.)

III. Delaunay's values for the portions of the form $ee'f(m)$ as given in vol. xxix. of the *Memoirs of the French Academy of Sciences* on pp. 804, 805, 818, 819, 822, 823, 838, 839, 845, 846.

IV. The values of the last terms of the series in powers of m calculated by Delaunay to obtain the values in III.

V. The approximate ratio of the last group of terms of the same sign in Delaunay's series in powers of m. (See below.)

VI. The differences between columns II. and III.

Column V. is of importance as showing how an estimate can be made for the neglected terms in Delaunay's series. The series for any coefficient in the longitude all appear to have a common property which seems to be worth emphasising. In any coefficient we can pick out the terms of the form

$$e^ie'^j\gamma^p(a/a')^q(\alpha_0+\alpha_1m+\alpha_2m^2+\ldots)$$

where

$$\alpha_0, \alpha_1, \alpha_2 \ldots$$

are the positive or negative numerical multipliers. Then, in

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nearly all cases, it will be found that (1) a series of the numbers,

$$a_0, a_1, a_2 \,.\, a_r,$$

are of the one sign, followed by another series,

$$a_{r+1} a_{r+2} \,.\,.\, a_s,$$

of the opposite sign, again followed by a third series,

$$a_{s+1}, a_{s+2} \,.\,.\,.\, a_t,$$

of the first sign, and so on; the first change of sign may occur anywhere, but there are generally five or six terms at least before the sign changes again. (2) Whatever law governs these series, the natural result follows that the numerical multiplier on either side of the new sign is very small. (See *Monthly Notices*, vol. lii. p. 74.) (3) The terms of the series forming a group of the same sign have approximately a common ratio. Hence, if one of Delaunay's series has an approximately common ratio for the later terms, and if, in addition, the last term given do not fall in with this ratio, but is very small, we can accept it as very probable that the next term will be of the opposite sign, and can estimate the rest of the series accordingly.

A single example will illustrate these points, and will explain the only apparent discordance between my results and those of Delaunay as given in the table above. This is the coefficient of the term with argument $4D-l+l'$. Expressed in seconds, this coefficient is—

Term in $ee'm^3$...	$-0''\cdot4750$
„ „ $ee'm^4$...	$-0\cdot2518$
„ „ $ee'm^5$...	$-0\cdot0178$
Sum	$-0\cdot7446$
My coefficient being	$-0\cdot5780$
Value of the rest of Delaunay's terms	$+0\cdot1666$

showing that the terms must change sign.

The most important results are those for the coefficients of the terms with arguments $l-l'$ and $2D-l+l'$. The results show that an estimate with the common ratio $\frac{1}{2}$ would have given nearly the correct result.

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